Counting the Numbers Fairly:
The Equal Proportional Satisfaction of Incommensurable Values

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I Taurek's Numbers Problem

It seems right that we should distinguish between five persons each suffering some given amount of pain *x* and one person actually suffering some amount of pain which is five times as large as *x*. It could well be that we are properly moved to do something about both of these, that is, to reduce both the numbers of those suffering *x* and to reduce the actual suffering from *5x* to something lower. But we would surely say that we would be doing something different in each case. In the former we would be reducing the incidence of suffering; in the latter we would be reducing suffering itself.

But suppose that we could not do both, but only one or the other. Would it matter which we did? It seems that the classical utilitarian is committed to being indifferent between the two possible outcomes because the sum total of pain reduction is the same in each. So it looks like utilitarianism is insensitive to the above-mentioned difference between the incidence of some amount of suffering *x* and the actual suffering of it. For some this is because utilitarianism does not take seriously the separation between persons.[1]

What would someone like John Taurek say? Of course, John Taurek is famous for arguing that adding to the number of persons who are suffering some given loss does not make a moral difference.[2] So he seems to be committed to being indifferent between one person suffering a loss and five persons suffering that *same* loss. Taurek says that a larger number of people suffering the loss does not make the loss any worse *as a loss*; what matters are losses *to a person*, and there is no loss to a person that is any larger simply in virtue of the fact that more persons are suffering it. This may be true. However, in Taurek's position there seems to be some insensitivity, comparable to what we saw in the classical utilitarian, to the possibility that the outcome where five suffer the loss is worse in some *other* respect. After all, more persons are suffering this loss and surely the (same) losses of these additional persons should count for something (why should they count for nothing?) even if they do not count as a greater loss.[3]

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[1] See, e.g., Rawls (1971), 25-27, and Nozick (1974), 33.

[2] Taurek (1977)

[3] How they should count for something, and whether this presupposes some sort of significance for aggregation, is an issue between Scanlon (1998) and Otsuka (2006).

But what would John Taurek say about our original choice between reducing from five to zero the *number* of persons each suffering a pain of *x* and reducing from 5*x* to zero the *quantity of suffering* of some one individual. If what matters are losses to a person (and, more particularly, if this is all that matters), then one would have thought that Taurek would choose to reduce the loss of the one from 5*x* to zero. Interestingly, however, Taurek does not commit himself to that result, although he does speak of a reduction of the larger individual pain as a more "natural" focus of concern. In addressing the one individual who is suffering the larger pain in comparison to the group of individuals each suffering a smaller pain, he says:

> It is not my intention to argue that in this situation I ought to spare you rather than them just because your pain is "greater" than would be the pain of any one of them. Rather, I want to make it clear that in reaching a decision in such a case it is natural to focus on a comparison of the pain you will suffer, if I do not prevent it, with the pain that would be suffered by any individual in this group, if I do not prevent it. I want to stress that it does not seem natural in such a case to add up their separate pains.[4]

What Taurek describes here as natural has come to be called "pairwise comparison".[5] Rather than add up the different quantities of pain or loss that different individuals might suffer, and then choose so that this sum is minimized, under pairwise comparison one engages in a series of comparisons between pairs of the different individuals involved and then chooses so that the loss *to any one individual* is minimized. This seems easy enough to understand and to effect, and it seems to attend directly to the concern that what really matters are losses to the person rather than aggregate losses that are the losses of no one in particular. Indeed, this would appear to be why Taurek refers to it as a "natural" way to think about the problem. Why then did he not endorse it as a logical extension of the argument that the numbers do not count when all losses are the same?

Perhaps he was uncomfortable with some "unnatural" implications of pairwise comparisons. For example, suppose that state of affairs A provides for utilities to three individuals, Tom, Dick, and Harriet (in that order) as follows:

A: (1, 2, 3)

It seems natural to think that the following state of affairs B, which provides for exactly the same utilities, albeit in a different order, is equally as good.

B: (3, 1, 2)

[4] Taurek (1977) 309

[5] Nagel (1979) 125

After all, what matters, typically, for an impartial observer (it might be different for a friend or relative of one of the individuals) is the utility information in the different states of affairs, not *whose* utilities they are. Call this invariance condition (namely, that our assessment of the two states of affairs does not vary with the names that are attached to the different utilities) *anonymity*. The problem is that in a decision between A and B, pairwise comparison indicates that B is the better choice. Tom, whose utilities are represented first, gains more in a choice of B over A (namely, 2 units) than either Dick or Harriet does in a choice of A over B (namely, 1 unit each). Or, to carry on with the idea of losses and what they mean to individual persons, Tom loses more in a choice of A over B than what either Dick or Harriet loses in a choice of B over A. Thus, pairwise comparison would have us choose B over A even though, under the anonymity condition, there is no real difference between the alternatives for choice. This seems odd.

Of course, pairwise comparison explicitly attends to the *gains* and *losses* of individuals as we move *between different alternatives*, a feature that it shares with the more aggregative versions of utilitarianism. So perhaps we should not be surprised that, on the basis of a comparison of these gains and losses for individuals, it recommends *choices* between alternatives when these alternatives, viewed merely as isolated *states of affairs* and from some impartial point of view, seem equally good (as they do, say, under anonymity).[6]

However, this dependence of choice on what is gained and lost for individuals across differently available alternatives carries another (related) problem with it. Consider, for example, the following possible alternative states of affairs, C, D, and E, which (as before) provide for utilities to Tom, Dick, and Harriet in the same order as before:

C: (6, 10, 2)

D: (1, 10, 9)

E: (8, 4, 3)

Here, the method of pairwise comparison, presented with the choice between C and D, would choose D; the loss to Harriet in moving from D to C (7 units) is larger than the loss to either Tom (5 units) or Dick (who experiences no loss either way) in moving from C to D. Analogously, pairwise comparison would choose E over D. Finally, and in violation of *transitivity*, it would

[6] See Lubbe (2008), at 74-75 for a discussion of how "better choices" can be importantly different from "better states of affairs". This contrast is also important to understanding what might be problematic in too quickly accepting certain choice consistency conditions like Sen's "contraction consistency" (see Otsuka 2004, at 420n17); such conditions, which call for consistency of choice as the opportunity set varies, tend to collapse the distinction between "choices" (defined by the set of available alternatives) and an assessment of states of affairs or alternatives for what they are (and independent of what other alternatives might be available for choice). Also see notes 7 and 16 below, and accompanying text.

recommend choosing C over E.[7] Thus, pairwise comparison cannot recommend *any* choice from the triple of alternatives without there being another alternative deemed better than it *on its own terms*. This too seems odd.[8]

So perhaps Taurek was wary of these unnatural implications of any "natural" extension of his innumerate ethics to a more ambitious and general system of pairwise comparison. Maybe he thought anonymity and transitivity were conditions worth holding on to. The problem with that interpretation of his view, as a number of his critics have pointed out in various ways,[9] is that the combination of anonymity, transitivity (at least in its strong form that requires transitivity of the "at least as good as" relation, that is, transitivity of social preference and indifference), and one further condition, the *Pareto* principle, commits him to saving the greater number of persons. As Taurek is at least reported to accept the Pareto principle (the principle, roughly, that would have us do something good for a person if we could do so without doing ill to another), this seems to be an embarrassing implication for his position, at least if he also wants anonymity and transitivity.[10]

But the situation is actually far worse than that. In what Taurek views as a natural concern for the greater individual pain or loss, what matters in any comparison between individuals is the *quantity* of pain or loss to the individual that we might prevent in choosing between the different alternatives, not the absolute level of pain that some individual might be suffering under some choice. So Taurek's concern for a given loss to an individual (in comparison to the loss of another individual) seems to be invariant to whether that loss is suffered by someone who is badly off or someone who is well off. In other words, the real issue is in the (interpersonally) *commensurable cardinality* of the welfare gains and losses for the individuals, not in any overall *ordinal* comparability, that is, not in any comparison of the *levels*

[7] Pairwise comparison will also choose E from the triple of alternatives (C, D, E), but C from the pair (C, E), a violation of Sen's contraction consistency condition; see above at n.6

[8] This violation of transitivity is predictable once one appreciates that pairwise comparison is the social choice rule that is logically analogous to the "minimax regret" decision rule in individual choice under uncertainty, an individual choice rule that has long been recognized as giving rise to intransitivity. See Milnor (1954) 49.

[9] Kavka (1979), Hirose (2004), Otsuka (2006). Roughly, these different proofs show that under the assumed conditions Taurek's decision rule would indicate (i) social indifference between X, saving Xavier, and Y, saving Yvonne, (ii) social indifference between X, saving Xavier, and Z, saving *both* Yvonne and Zak (because the numbers don't count), and yet (iii) a strict social preference for Z, saving both Yvonne and Zak, to Y, or saving Yvonne alone (because of Pareto). But if Z is socially preferred to Y, and Y is socially indifferent to X, then (by transitivity) Z should be socially preferred to X, not indifferent to it (i.e., the numbers should count).

[10] Kamm (1993) at 97n12 reports Taurek's agreement with the Pareto principle.

of welfare that different individuals might reach (or not reach) with these gains and losses.[11] Again, this is a feature of Taurek's approach to the problem of choice that he shares with the more aggregative versions of utilitarianism, which are also focused on possible gains and losses of utility as we move between alternatives, and how these compare across individuals. Taurek appears to accept this, and only resists the further idea that it makes any normative sense under those utilitarian approaches to add up these separate gains and losses across individuals into a sum total. According to Taurek, that is nonsensical because it separates the measure of losses from the persons whose losses they are.

However, Taurek's "natural" (and exclusive) focus on the *commensurable cardinality* of utility (avoidance of pain, pleasure, welfare, or any such good) across individuals, when this is combined with *anonymity*, *transitivity*, and the *Pareto* principle, commits him (at least in an otherwise welfarist and consequentialist framework) to classical utilitarianism wherein the chooser maximizes the sum total of utility or good, the very sort of thing that Taurek labels as nonsensical. To see this, consider the following two choices (again for Tom, Dick and Harriet):

F: (1, 3, 3) (sum total of utility = 7)

G: (4, 1, 1) (sum total of utility = 6)

If the sum total of utility is the indicator of preferred choices, then F should be socially preferred to G (or, in symbols, F > G). Suppose that this was not true, i.e., suppose that *not*-(F > G). It is easy to show that this will lead to a contradiction of at least one of our conditions: transitivity, anonymity, the Pareto principle, and the commensurable cardinality of utility across persons.[12] The proof begins by considering the following sequence of paired choices:

F: (1, 3, 3)

H: (1, 1, 4)

By anonymity, H is socially indifferent to G (being a mere permutation of its payoffs). Therefore, by transitivity, if *not*-(F > G), as assumed, then *not*-(F > H). Now consider the next pair.

[11] Some proponents of pairwise comparisons are sensitive to both the units (gains and losses) and the levels of different persons' welfare. In Nagel (1979), for example, a person's individual claim against a larger number of persons might be particularly salient either because he is very badly off (even though the claim is quantitatively small) or because the one person's claim is quantitatively large (even though he is not badly off). Exactly how these different sorts of concern for an individual's claim are integrated in such models of pairwise comparisons is unclear.

[12] The following proof is from Chapman (2010).

I: (1, 3, 1)

J: (1, 1, 2)

In I (as compared to F), and in J (as compared H), all that is changed is that (a constant) 2 cardinal units have been subtracted from the third individual's good in each case; so the gain (or loss) in moving between these two alternatives (as significant and comparable for all persons) has been preserved. Therefore, by commensurable cardinality, if *not*-(F > H), then *not*-(I > J). Finally, consider the pair:

K: (1, 1, 3)

J: (1, 1, 2)

By anonymity, K is socially indifferent to I (again, being a mere permutation of its payoffs). Therefore, if *not*-(I > J), then *not*-(K > J). But, by the Pareto condition, K > J, a contradiction. So (to avoid this contradiction under these conditions) it must be that we have started out with an incorrect presupposition in *not*- (F > G). Therefore, F > G, which is exactly what classical utilitarianism recommends.

It is easy to see that the same method of proof can be trotted out for *any* possible pair of choices where the total utility or good in one choice is larger than the total utility or good in the other. Effectively, by the repeated (and transitively linked) application of anonymity and commensurable cardinality, any larger total of good can be reduced, finally, to a Pareto comparison. Utilitarians, and other like-minded proponents of sum totals of the good (however construed, e.g., as utility, preference satisfaction, pleasure, or the relief from pain), will no doubt be pleased. Taurek, however, would seem to have some re-thinking to do, at least if he is tempted to accept transitivity, anonymity, commensurable cardinality, and the Pareto principle. While none of these conditions appears, on its own, to be an aggregative condition, together they combine to rank alternative choices according to their sum total of good, the very sort of aggregation that he would deem ethically nonsensical.

However, Taurek might take some comfort from the fact that, along the way to its result, the proof shows exactly what he is worried about. Specifically, the proof shows that while the moral significance of gains and losses of some utility or good are preserved *inter*personally, or across individuals, they are not preserved in their significance *intra*personally, that is, or for any one individual. Commensurable cardinality appears to begin with some basic sensitivity to an individual's gain or loss (this is what the cardinality of the numbers attends to), and then simply adds the further idea that this individually significant gain or loss needs to be commensurate with the individually significant gains and losses of other persons (so that the cardinality becomes commensurable cardinality). Indeed, this much also seems to be assumed in pairwise comparison, which is very much focused on an across-person comparison (or commensurability) of what are, nonetheless, also individually significant gains and losses (cardinality). But when commensurable cardinality is combined with anonymity and transitivity, this apparent sensitivity to individually significant gains and losses disappears.

We can see this if we compare the choice between F and G with the choice between F and H. Under anonymity, G and H cannot be distinguished and so must be ranked equally as social choices. So, by transitivity, whatever social ranking holds between F and G must also hold between F and H. But, *intra*personally, what is at stake for Tom at the first bracketed position in the choice between F and G, a difference of 3 units of the good, has changed dramatically in the choice between F and H, where, for Tom, there is no longer any difference at all. Of course, that *overall* 3 unit advantage for G has been "preserved" in H, *inter*personally, as the conversion (at the third bracketed position in the choice) of a 2-unit advantage for Harriet in *F over G* into a 1-unit advantage for Harriet in *H over F* (an actual preference reversal for Harriet). So, in this sense, the *inter*personal significance of the cardinality has been preserved even though the *intra*personal cardinal significance of the numbers for Tom and Harriet (and, for Harriet, even their intrapersonal *ordinal* significance) is lost. *It is as if there is morally significant aggregate good here that can be completely detached from, and exist prior to, the good of any individuals.* Of course this aggregate good must supervene on the good of individuals in that there cannot be changes in the aggregate good without there also being changes at the individual level. (The economic theorist captures this idea, typically, by insisting that the social welfare function, or social ordering, be a positive function of the welfare of each and every individual.)[13] But in merely supervening on individual good, the aggregate good, measured in this additive way, need not make any *sense* of any individual's good. This does sound like the very thing that worries Taurek.

Now, focusing on the early stages of the above proof (the ones involving alternatives F, G, and H) might suggest that the problematic defining conditions for classical utilitarianism are anonymity and transitivity. After all, the last paragraph has suggested a problem at these early stages before any of the other conditions are even mentioned. But I want to suggest that Taurek's real problem is with the idea of (interpersonally) *commensurable* cardinality (something that might also explain his puzzling reluctance to endorse even pairwise cardinal comparisons). If he is prepared to relax that condition to *incommensurable* cardinality, then it is easy to show that he can hang on to anonymity, transitivity, *and* the Pareto principle, all without being classically utilitarian. Moreover, by denying the interpersonally commensurable significance of cardinality while holding on to its strictly intrapersonal significance, he continues to operate with the quantities of personal loss that seem to move him without lending ethical significance to an impersonal space of losses that, he claims, resonates for no one person in particular.

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[13] Cf. Hirose (2004) 66

Of course, all this would be mere wishful thinking if there was no way to combine anonymity, transitivity, Pareto, and incommensurable cardinality. Fortunately, however, we have a possibility theorem for this combination of conditions readily at hand in the form of John Nash's famous arbitration solution to the bargaining problem.[14] Moreover, while Nash provides a uniqueness result on these conditions (his solution being the only method of choice that satisfies all these conditions), his way of thinking about the possibility of maximizing over an informational domain based on the incommensurable cardinality of values opens up (under slightly different conditions) a whole new range of approaches to aggregation that even John Taurek might want to explore. Some of these approaches are already active (albeit in a somewhat unselfconscious way!) in legal thinking under the idea of proportionality.[15] The next two sections offer a closer look at these different approaches and what they might suggest as a solution to the original numbers problem posed by Taurek.

II Maximizing Over the Proportional Satisfaction of Utility or Value

As a solution to the bargaining problem, John Nash proposed that the arbitrator maximize the (multiplicative) *product* of the individuals' utility gains over the status quo. (Given that the gains are not interpersonally commensurable, the status quo can be normalized under *separate*, or person-specific, linear transformations of the individual utility scales into a point of zero utility for each and every individual, effectively reducing all the gains above the status quo to more familiar utility numbers.) As an aggregation rule, Nash's solution will generally recommend different choices from either classical utilitarianism or pairwise comparison. So, for example, in the choice between the alternatives C, D, and E above, Nash's product rule would recommend C as best, classical utilitarianism (which maximizes the additive sum) would choose D, and (as earlier argued) pairwise comparison would choose E.

The insensitivity of the Nash solution to any commensurability of these cardinal utility measures can be appreciated once one sees that a heavier weighting of any one individual's utility scale (e.g., a doubling of Tom's first place utility numbers in the alternatives C, D, and E, something that would change the recommendation of classical utilitarianism, relying as it does on commensurable cardinality, from choosing D to choosing C) would only rescale the product of the utilities of each alternative (e.g., doubling the product of the numbers in each row), but otherwise leave the social ordering of the alternatives unchanged.

[14] Nash (1950)

[15] For discussion of the linkage between the legal idea of proportionality and the sorts of aggregation rules that are more commonly discussed in social choice, see Chapman (2010), (2011)

Of course, we did not set up the alternatives C, D, and E as possible choices within an arbitration exercise on a bargaining problem. But Nash's solution need not be limited to bargaining, and the measure of utility gains need not be from some status quo from which bargaining naturally begins. The state of zero utility could in principle be any alternative state of affairs (real or hypothetical) from which it makes sense to measure an individual's utility gains, and the interval scale, required for a measure of cardinality, could be determined by some ideal point which sets the maximum possible gains for that individual.

Moreover, the invariance of the Nash solution to separate (person-specific) linear transformations of the utility numbers (again, invariant because of the interpersonal incommensurability of these cardinal numbers) allows us to rescale the numbers for each individual for each alternative between zero and the ideal point on a hundred point scale, something that would allow us to think of the numbers as the proportional (or percentage) satisfaction of the individual's ideal that is provided by each alternative for choice. This is helpful because thinking about these Nash numbers as showing the *proportional* satisfaction of each individual's utility scale allows us to appreciate quite naturally that the cardinal numbers have no interpersonal significance. We are simply not tempted to think of proportionality comparisons in this way. Suppose, for example, that both your barrel and my barrel were drained of one third of their content of water, that is, they were subject to an equal proportional draining. We would not be tempted to say that you had lost the same quantity of water as I had. To know that, we would need to know something about the relative size (or commensurability) of your barrel as compared to mine. If your barrel was twice as large, then you would have lost more; if half as large, you would have lost less. But our judgment of an equal proportional loss would be invariant to either of these possibilities; a proportionality comparison does *not* depend on commensurability.

What can be said of barrels can equally be said of the individual utility numbers in the Nash solution. While they provide significant measures of the losses, or the impact on cardinal utility, that the different social choices impose on different individuals, they say nothing about any impersonal measure (or commensurability) of that impact. This would appear to be the sort of thing Taurek might be looking for. And he can have it, under the Nash solution, without sacrificing anonymity, transitivity, or the Pareto principle.[16]

Nevertheless, we might still worry that Taurek would be unmoved by a product of proportional individual utility gains, even if these proportional utility gains do not presuppose

[16] Nash's (1950) axiomatization of his bargaining solution is explicit in requiring anonymity and the Pareto principle (as well as incommensurable cardinality). Rather than require transitivity (a property of the social preference relation) he required the choice theoretic equivalent of "contraction consistency" (of inclusion and exclusion); see above, notes 6 and 7. Arrow (1959) and Sen (1971), (1977) have shown that there is a logical equivalence between these preference theoretic and choice theoretic conditions.

any commensurability in some (for Taurek, meaningless) impersonal space. Nash's product rule begins with individually significant measures of loss and gain, and, unlike what we saw in the additive forms of aggregation (as evidenced in the above proof of classical utilitarianism), it seems to do no violence to these individually significant measures as it aggregates. Still, one might wonder, with Taurek, about what the public significance of a *product* of proportional individual utility gains really is. When we multiply the length, width, and height of a box, and thereby determine its volume, we know that our product has come up with something that matters in an independently significant three dimensional space. From this we can determine how much we can put in the box, for example.[17] But if we have denied the significance of any cardinal commensurability of utilities, then have we not also denied the public (aggregative) significance of the impersonal space in which the product (as much as the sum) of those utilities might resonate?

If there is a problem here, it could be addressed by adopting a different sort of maximand over these proportionality comparisons. We could, for example, maximize the satisfaction of proportional satisfaction of any *one* individual's utility. There would be nothing aggregative about that! And, in a way, this is what we see when a judge at a dog show chooses the "best in show". She looks over all the different champions of each breed class and then chooses that one dog which, *on its own breed scale*, is proportionally the best. ("This is a better Schnauzer on the Schnauzer scale than that Great Dane is on its Great Dane scale.") Such comparisons do not presuppose any commensurability across breeds, nor do they contemplate some independent conception of one "ideal dog" with respect to which the different breeds are deficient to varying degrees. Moreover, different judges, who might have different preferences over the different breeds, should, at least in theory, all come to the same proportional judgment as to which dog is best; as the idea of incommensurability between breeds suggests, their judgments will be invariant to the different "weights" that each judge might be tempted to put on the different breeds.

Further, we could extend this idea to a judgment of which dog show, each with many dogs on display, was the best. It would be that dog show for which the "best in show" dog was the best overall (across all the different dog show winners) on the basis of this same individualized proportional comparison (a kind of "maximax" rule). And if the top dogs in any two dog shows were tied, we could go on to compare, on the same proportional basis, the two next best dogs in each show to break that tie, and so on (a kind of "leximax" extension of maximax).

However, a familiar problem begins to show itself at this point. The maximax rule (and even its leximax extension) concentrates completely in its comparison between alternatives on

[17] I am grateful to Michael Moreau for this example.

the one dog that is best in the show. So we could have a dog show with a large number of very fine dogs on display that would nevertheless finish second to the one dog show, with many fewer fine dogs, simply because the best in show in the latter was better than the best in show in the former (a kind of "positional dictatorship" result in social choice theoretic terms). The attractive feature of the Nash product rule over the maximax rule is that it *is* more aggregative, that is, that it does allow us to consider the relative proportional satisfaction of more of the values or utilities at stake in the choice. Perhaps Taurek should be moved by the idea that the Nash product rule allows more individuals to make a difference with their *individually* significant utilities. This does not require us to concede any significance to some (for Taurek) meaningless impersonal space of utility. That idea was left behind once we adopted the framework of proportionality comparisons.

Taurek might still argue, however, for a quite different sort of maximand. He might concede that we need to find a way to attend to the claims of more of the individuals than what the highly individualized maximax rule allows. But he might still resist the idea that attending to the claims of "more *of* the individuals" can sensibly be reduced to the idea of attending to the claims of "more individuals". The latter, he might suggest, just smuggles in the idea that the numbers count, even if it does not presuppose some meaningless impersonal space in which the claims of the greater numbers can resonate as such. So he might propose that we let in a consideration of the utility claims of other individuals as follows: Maximize (so far as possible) the *equal* proportional satisfaction of individual utilities, that is, maximize the proportional satisfaction of any one person's utility so long as we can also achieve the same (or closest to the same) level of proportional satisfaction of utility for any other person.

Arguably, this is what law courts do when, in the context of a choice between values rather than individual utilities, they attend to the proportional satisfaction or impact of different possible decisions on quite different (incommensurable) legal values (usually some state interest on the one hand and some individual's constitutional right on the other). Like the Nash product rule (and unlike the maximax dog show rule), this maximand obliges us to attend to the proportional satisfaction of more than one person's (albeit anonymously considered) utility. But, unlike for the Nash rule, the obligation to equality could mean that the Pareto principle would be violated, since the greater proportional satisfaction of an individual utility that was already at the highest level of proportional satisfaction would increase the inequality of proportional utility satisfaction if the proportional satisfaction of the other individual utilities were themselves unchanged (and, even if also changed for a higher level of satisfaction, were not changed proportionally as much).

However, despite the possible violation of the Pareto principle this equality maximand *does* attend to the public significance of proportional utility satisfaction and it does so in a way that Taurek might accept. In holding the proportional satisfaction of any one person's utility hostage to the (equal, or closest to equal) proportional satisfaction of any other persons' utility, the satisfaction of each person's utility has an obvious significance for the satisfaction of every other person's utility. This is a form of public significance in the satisfaction of utilities, although it is not the sort of *impersonal* public significance of utilities that we see under commensurability. Here the public significance is more *interpersonal* than impersonal (or what Stephen Darwall has called *second-personal* rather than *third personal*[18]) in that the equality relation holds directly between the utilities and is not (because under proportionality measures it could not be) mediated by some measure of relative weights provided by commensurability within some third space. Under an equal proportional utility satisfaction requirement, the satisfaction of each person's utility is held accountable to, *and only to*, the (equal) claim that any other person has to have his or her utility proportionally so satisfied.[19]

So far this has largely been a brief for Taurek (or, at least, an interpretation of what he really needs for his argument) and critics of Taurek will fast be turning impatient. Have we not, under equal proportional utility maximization, ended up with the same sort of number insensitivity that Taurek originally proposed? Some might say that this is the problem, not a solution! Indeed, isn't equal proportional utility satisfaction even worse than anything Taurek proposed in that Taurek (as noted above) seems to support the Pareto principle and equal proportional utility satisfaction might not?

Perhaps, but in working through the claims of individual utility on social choice in only an intrapersonally (and, ultimately, second-personally) significant way, I hope to have taken Taurek's concerns about the possible meaninglessness of impersonal comparisons, and aggregations based on those comparisons, as seriously as I can. And now I want to use the more general method of equal proportional value (not just utility) satisfaction to provide a quite different sort of solution to Taurek's original problem where we must choose between saving either the one person or the five persons from death. As we shall see, this solution does take seriously the idea that there is some greater good in saving the greater number, but it does so in a way that balances that good against the fairness that Taurek saw in giving everyone an equal chance of being saved. However, consistent with the argument of this paper so far, and in the spirit of Taurek's skepticism about impersonal spaces of evaluation, it balances, or brings into equality, only a proportional satisfaction of these two values, viz., the value of saving life and the value of fairly allocating the chance of being saved. As we shall see, the consequence of such an approach is a solution to the numbers problem that is much closer to Taurek's solution of equal chances for all than what others have proposed.

[18] Darwall (2006)

[19] It is the interpersonal significance of an equal proportionality assessment that makes it particularly suitable as a content for legal processes, where the different parties to the litigation are holding each other accountable on the basis of claims and counterclaims, and third parties have only very limited standing to have their interests addressed. On this, see Chapman (2011) and (2012).

III Equal Proportionality and the Weighted Lottery

Since John Broome first suggested (but did not endorse) the idea in 1984, a weighted lottery has often been thought to provide an attractive compromise between the two competing values that are at stake in Taurek's choice problem.[20] On the one hand there is the value that one choice provides in allowing more people to avoid some loss, say, the loss of their individual lives should some drug not be provided to them. This seems conducive to the value or *good* of saving lives or avoiding losses where we can. On the other hand, there is (for Taurek and for others too) the idea that flipping a fair coin between the two (differently numbered) groups of persons gives each and every person an *equal maximal chance* to survive or avoid the loss. The *fairness* in that also seems to be something of value.

One way to give some recognition to both of these very different sorts of values is to have a weighted lottery determine the choice between the two differently sized groups. Unlike an equal chance lottery, a weighted lottery, at least if the probability of saving is weighted in favour of the larger group, seems to allow for some accommodation of the greater numbers that could be saved from the loss while at the same time giving those in the smaller group some chance to avoid the loss themselves.

Typically, the probabilities in the weighted lottery are determined by the relative sizes of the two groups of persons on the basis of something called "the principle of proportional chances". So, in a choice between saving the one and saving the five, the weighted lottery would save the great number with a probability of five sixths and would save the one with a probability of one sixth. Such a principle allows each person's possible loss to have the same proportionate claim on the lottery and its probabilities. This seems fair, and yet it also holds out a greater possibility of saving the larger number, something which is good.

Different proponents of the weighted lottery idea have different arguments for getting to this principle of proportional chances, but all of them seem to depend on conjoining (at some point, sometimes in a second stage of a multi-stage process[21]) a concern for doing good, or for avoiding waste (e.g., in not saving someone we could also save without incurring any additional cost), with a concern for achieving fairness. Under these sorts of arguments, this conjoining of the two values, good and fairness, is determined by the lottery itself. We are to begin with fairness (under a very particular interpretation, viz., that each person begin with the same $1/n$ chance of being saved where n is the total number of persons involved) as the prior value (so that no individual can complain that the lottery did not begin fairly) and make any adjustment for the

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[20] Broome (1984). See also Kamm (1993), Timmermann (2004) and Saunders (2009).

[21] See, e.g., Timmermann (2004) and Saunders (2009)

possibilities of achieving some greater good as a matter of luck (e.g., if someone in one persons' group has randomly been chosen to be saved, then others in that person's group can now be saved as well at no additional cost).

But we could be more systematic than this in our accommodation of fairness and the good and, further, begin with the idea (proposed by Taurek, and supported by Hirose[22]) of maximal fairness rather than some truncated version that allows (unfairly?) for a lower level of equal chances to be arbitrarily corrected as a matter of luck, something that (too predictably) seems to benefit the larger numbered group.[23]

Consider again the example where we must choose between saving one person and five persons, the original choice proposed by Taurek for our consideration. Given the unfortunate choice in the matter (in that we cannot save all), the maximum good that we could do is to save five. Any lottery we adopt would have, as its *expected* value for saving persons, some number that is only some proportion of that maximal value. We would measure that expected value in the usual way as:

$(p \text{ x } 1) + (1\text{-}\ p) \text{ x } (5)$, where p is the probability of saving the one person.

So the *proportion of (maximum) good* (of life saving) achieved under the lottery would be:

$$\frac{(p \text{ x } 1) + (1\text{-}\ p) \text{ x } (5)}{5}$$

Analogously, the maximum fairness that we might achieve under the lottery would be to give everyone in the two groups the same fifty percent (or .5) chance of being saved. Any lottery we would adopt would provide for some p for saving the one which, again, would be some *proportion of (maximum) fairness*, or $p/.5$ (where $0 < p < .5$).

Now suppose that we wanted to systematically maximize the *equal* proportional satisfaction of these two very different values. What would our weighted lottery look like? To determine this we need to set the proportional satisfaction of good *equal to* the proportional satisfaction of fairness, and then solve for the probability p, i.e.,

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[22] Hirose (2007). For a reply to Hirose's critique of proportional chances, see Saunders (2009) 287.

[23] One more systematic way for coming up with the proportional chances lottery is to apply the Nash solution to the expected utilities of the six persons who are subject to the weighted lottery. The probability (of saving the one as opposed to the five) that maximizes the Nash product of the expected utility gains of the six persons is 1/6. Of course, with incommensurable utilities, it is still puzzling what public significance this Nash product of utility gains could possibly have. I am grateful to Joe Heath for pointing out that the Nash solution has this solution in this example.

Let $\frac{(p \times 1) + (1-p) \times (5)}{5} = \frac{p}{.5}$

Therefore, solving for *p*, we would determine that *p* = .357

So, the weighted lottery under equal proportional satisfaction of the two values, fairness and the good of saving lives, would have us save the one person with a probability of .357 and the group of five persons with a (complementary) probability of .643. The proportional satisfaction of each value (the amount of expected good as measured against the ideal of 5 persons saved, and the probability of the one being saved as measured against an ideal maximum fairness of a 50 percent chance) is the same at 71 percent (i.e., the proportion that 3.57 expected persons saved is of 5 and that a probability of .357 is of .5 respectively).

It is worth noting that this weighted lottery is significantly more favourable to the one person whose life might be saved than the more commonly accepted proportional chances lottery (which would only assign a one sixth or (roughly) a 17 percent chance to the one of being saved). This reflects the systematic and equal (proportional) attention that is given to Taurek's *maximum fairness* consideration from the beginning. As the arguments of both Timmermann and Saunders suggest, the proportional chances lottery begins with less than maximum fairness and then lets the lottery itself enhance the chances of those in the larger group as a matter of luck and a concern for a costless saving of additional lives. One can easily imagine Taurek balking both at where this argument begins (with less than maximum fairness) and how it makes further adjustments (on the basis of saving more lives where luck allows).

On the other hand, the equal proportional satisfaction lottery assigns a significantly *lower* probability to the chance of saving the one than Taurek does. This is, of course, because it assigns the same proportional concern to the *good* of saving lives as it does to Taurek's exclusive concern for achieving maximum fairness.

It is also worth noting how little the equal proportional satisfaction lottery changes with changing numbers in the larger group. For example, in a choice between saving one person and saving ten (rather than five), the probability that is given to saving the one is only reduced from .357 to .344 (with the equal proportional satisfaction of both the good of saving lives and fairness now reduced from 71 percent to around 68 percent). In a choice between saving one and one hundred, the probability of saving the one is about .334, or about the same as for a choice between one and ten (with roughly the same proportional satisfaction of the two different values). So, the additional numbers in the larger group very quickly do not count for much, even though we have introduced (in contrast to Taurek) an equal proportional concern for the good of saving lives. But this simply follows from the arithmetic of proportionality: as the numbers in the larger group increase, the additional good that is done by saving one more person *as a proportion* of the maximum good that might be done if we saved the larger group correspondingly decreases. So we should expect that, while the additional numbers in the larger group will count, they will count in a rapidly diminishing way. Indeed, as these numbers suggest, we quickly converge on assigning a probability of about one third to saving the one person regardless of the additional numbers that we add to the larger group.[24] This is a feature of the equal proportional satisfaction lottery that Taurek might well approve even if the lottery, unlike Taurek, does concede the greater good of saving more lives.

Of course, if the two groups that we might save are equal in size, then the equal proportional satisfaction lottery would assign equal probabilities to saving each group. This seems right. However, if the equally sized groups are small (say, one person in each group), then adding another person to one of the groups will change the probabilities more dramatically than if the two equally sized groups start out being very large. For example, in a choice between saving one person and two persons, the equal proportional satisfaction lottery would reduce the chance of saving the one person (in the one versus one scenario) from 50 percent to 40 percent. However, in a choice between saving 1000 persons and 1001 persons, the chance of saving the smaller group (with one less person) would hardly be reduced at all (going from 50 percent to 49.97 percent). This accords with the view advanced by Iwao Hirose when he discusses what he calls "large scale rescue cases".[25] However, the calculation under the equal proportional satisfaction lottery does not depend, as Hirose's does, on any notion of an aggregate unfairness that is larger because of the larger numbers of persons in the smaller group in these sorts of cases. In the equal proportional satisfaction lottery the unfairness is an unfairness *to each person* in the smaller group and it is number insensitive; it is a constant function of the proportion that the probability of saving the persons in the smaller group is of the maximum fairness that one can do by flipping a fair coin and giving everyone an equal 50 percent chance. This insensitivity of fairness to numbers is also a Taurek-friendly feature of the equal proportional satisfaction lottery.

IV Conclusion

In this paper I have attempted to provide an interpretation of Taurek's number insensitivity that also accounts, perhaps, for his reluctance to embrace the more ambitious and general method of pairwise comparison. I suggested that Taurek might have been worried about some of the implications of pairwise comparison for other attractive conditions for social choice like anonymity and transitivity. However, I have also shown that if Taurek is committed to

[24] This is easily calculated from a generalized version of the equation on the previous page. For the general case of n persons in the larger group (and one individual in the smaller group), $p = n/(3n-1)$. Thus as n tends to infinity p tends to 1/3.

[25] Hirose (2004) 77-79

anonymity and transitivity, and also (as he is reported to be) the Pareto principle and what he finds "natural" in commensurable cardinality, then he has committed himself to following the recommendations of the very same classical utilitarianism that he finds so nonsensical.

I proposed that Taurek consider relaxing commensurable cardinality to the sort of incommensurable cardinality that we observe in proportionality comparisons. This would allow him to give sense to the cardinal impact of different social decisions on individuals without committing to any impersonal cardinal significance in those decisions. I identified John Nash's famous product rule as one social decision rule that has the required informational base in incommensurable cardinality, or proportionality, although I suggested that Taurek might worry about the moral significance of a multiplicative product of utilities as much as he did about an additive summation of them. In response to that concern I suggested two other proportionality maximands, one of which, the maximax rule, seemed inadequately aggregative, and another of which, the maximization of equal proportional satisfaction, seemed to attend to the public significance of utility satisfaction in a more interpersonal and less impersonal way. I suggested that Taurek should be able to endorse the latter.

Finally I adapted the equal proportional satisfaction rule to the idea of a weighted lottery and showed how the probabilities of saving the different sized groups in a Taurek choice problem would vary under the equal proportional satisfaction lottery from those proposed under a more conventional proportional chances lottery. Generally, the probabilities of saving the smaller number are more generous under the equal proportional satisfaction lottery, and less sensitive to variations in the numbers in the differently sized groups. As a consequence I suggested that even though the equal proportional satisfaction lottery does introduce a systematic concern for the good of saving the greater number, as well as fairness, it does so in a way that is more Taurek-friendly than other weighted lottery proposals.

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